Eur. Phys. J. C 38, 373–378 (2004)
Digital Object Identifier (DOI) 10.1140/epjc/s2004-02040-y

THE EUROPEAN
PHYSICAL JOURNAL C

Stress-energy tensor for parallel plate on background of conformally flat brane-world geometries and cosmological constant problem

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Received: 17 March 2004 / Revised version: 29 September 2004 /
Published online: 26 November 2004 –

Abstract. In this paper, we calculate the stress-energy tensor for a quantized massless conformally coupled scalar field with a background of conformally flat brane-world geometries, where the scalar field satisfies Robin boundary conditions on two parallel plates. In the general case of Robin boundary conditions formulae are derived for the vacuum expectation values of the energy-momentum tensor. Further the surface energy per unit area is obtained. As an application of the general formulae we have considered the important special case of the AdS_{4+1} bulk; moreover the application to the Randall–Sundrum scenario is discussed. In this specific example for a certain choice of Robin coefficients, one could make the effective cosmological constant vanish.

1 Introduction

The cosmological constant was first introduced by Einstein in order to justify the equilibrium of a static universe against its own gravitational attraction. The discovery of Hubble that the universe may be expanding led Einstein to abandon the idea of a static universe and, along with it, the cosmological constant. However, the Einstein static universe remained of interest to theoreticians since it provided a useful model to achieve better understanding of the interplay of spacetime curvature and of quantum field theoretic effects. The recent years have witnessed a resurgence of interest in the possibility that a positive cosmological constant Λ may dominate the total energy density in the universe [1–3]. At a theoretical level Λ is predicted to arise out of the zero-point quantum vacuum fluctuations of the fundamental quantum fields. Using parameters arising in the electroweak theory results in a value of the vacuum energy density $\rho_{\mathrm{vac}} = 10^6\,\mathrm{GeV}^4$ which is almost 10^{53} times larger than the current observational upper limit on Λ which is $10^{-47}\,\mathrm{GeV}^4 \sim 10^{-29}\,\mathrm{g/cm}^3$. On the other hand the QCD vacuum is expected to generate a cosmological constant of the order of $10^{-3}\,\mathrm{GeV}^4$ which is many orders of magnitude larger than the observed value. This is known as the old cosmological constant problem. The new cosmological problem is to understand why ρ_{vac} is not only small but also, as the current observations seem to indicate, is of the same order of magnitude as the present mass density of the universe.

In recent years, there has been hope to understand the vanishing cosmological constant in extra dimensional theories [4–15]. It is generally believed that fine-tuning is necessary for a very small cosmological constant in 4-dimensional theories [16–18]. This leads one to search for a naturally small cosmological constant in higher dimensional theories. However, for the usual compactification of a higher dimensional theory to an effective 4-dimensional theory, one ends up with a normal 4-dimensional theory, and the fine-tuning problem generically reappears. This is the case for the usual Kaluza–Klein (KK) compactification, and for the generic compactification with large extra dimension [19]. The Randall–Sundrum (RS) model [19] provides the hope of avoiding this pathology. This higher dimensional scenario is based on a non-factorizable geometry which accounts for the ratio between the Planck scale and weak scales without the need to introduce a large hierarchy between the fundamental Planck scale and the compactification scale. The model consists of a spacetime with a single S^1/Z_2 orbifold extra dimension. Three-branes with opposite tensions reside at the orbifold fixed points, and together with a finely tuned negative bulk cosmological constant serve as sources for 5-dimensional gravity.

In the present paper we will investigate the vacuum expectation values of the energy-momentum tensor of the conformally coupled scalar field with a background of conformally flat brane-world geometries. We will consider the general plane-symmetric solutions of the gravitational field equations and boundary conditions of the Robin type on the branes. The latter includes the Dirichlet and Neumann boundary conditions as special cases. The Casimir energy-momentum tensor for these geometries can be generated from the corresponding flat spacetime results by using the standard transformation formula [20, 21]. Previously this method has been used in [20] to derive the vacuum stress on parallel plates for a scalar field with Dirichlet boundary conditions in de Sitter spacetime and in [21] to investigate

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the vacuum characteristics of the Casimir configuration on a background of conformally flat brane-world geometries for a massless scalar field with Robin boundary conditions on plates. Also this method has been used in [22] to derive the vacuum characteristics of the Casimir configuration on a background of static domain wall geometry for a scalar field with Dirichlet boundary condition on plates (for investigations of the Casimir energy in brane-world models with dS branes, see [23–28]). For Neumann or more general mixed boundary conditions we need to have the Casimir energy-momentum tensor for the flat spacetime counterpart in the case of the Robin boundary conditions with coefficients related to the metric components of the brane-world geometry and the boundary mass terms. The Casimir effect for the general Robin boundary conditions on a background of the Minkowski spacetime was investigated in [29] for flat boundaries, and in [30, 31] for spherically and cylindrically symmetric boundaries in the case of a general conformal coupling (for Robin-type conditions see also [32, 33])[1]. Here we use the results of [29] to generate the vacuum energy-momentum tensor for plane-symmetric conformally flat backgrounds; in the Sect. 2 we review this work briefly. Further in Sect. 3 the surface energy per unit area which is located on the branes is obtained. This surface term is zero for Dirichlet or Neumann boundary conditions but yields a non-vanishing contribution for Robin boundary conditions. In the general case (general coupling), the stress-energy tensor diverges close to the branes. This would also be expected in the conformal case if the branes are curved [34]. In Sect. 4 the important special case of an AdS background is considered, and we obtain an explicit relation between the cosmological constant of the AdS_{4+1} bulk and the brane tension (which is the surface energy per unit area located on the branes). Next, the application to the Randall–Sundrum is discussed. Finally, the results are listed and discussed in the last section.

2 Vacuum expectation values for the energy-momentum tensor

In this paper we will consider a conformally coupled massless scalar field $\varphi(x)$ satisfying the equation

$$(\nabla_\mu \nabla^\mu + \xi R)\,\varphi(x) = 0\,, \quad \xi = \frac{D-1}{4D}\,, \tag{1}$$

with a background of a $D+1$-dimensional conformally flat plane-symmetric spacetime with the metric

$$g_{\mu\nu} = \mathrm{e}^{-2\sigma(z)}\eta_{\mu\nu}\,, \quad \mu,\nu = 0,1,\ldots,D\,. \tag{2}$$

In (1) ∇_μ is the operator of the covariant derivative, and R is the Ricci scalar for the metric $g_{\mu\nu}$. Note that for the metric tensor from (2) one has

$$R = D\mathrm{e}^{2\sigma}\left[2\sigma'' - (D-1)\sigma'^2\right]\,, \tag{3}$$

where the prime corresponds to differentiation with respect to z.

We will assume that the field satisfies the mixed boundary condition

$$(a_j + b_j n^\mu \nabla_\mu)\,\varphi(x) = 0\,, \quad z = z_j\,, \quad j = 1,2\,, \tag{4}$$

on the hypersurfaces $z = z_1$ and $z = z_2$, $z_1 < z_2$; n^μ is the normal to these surfaces, $n_\mu n^\mu = -1$, and a_j, b_j are constants. The results in the following will depend on the ratio of these coefficients only. However, to keep the transition to the Dirichlet and Neumann cases transparent we will use the form (4). For the case of the plane boundaries under consideration, introducing a new coordinate y in accordance with

$$\mathrm{d}y = \mathrm{e}^{-\sigma}\mathrm{d}z\,, \tag{5}$$

the conditions (4) take the form

$$\begin{aligned} &\left(a_j + (-1)^{j-1} b_j \mathrm{e}^{\sigma(z_j)}\partial_z\right)\varphi(x) \\ &= \left(a_j + (-1)^{j-1} b_j \partial_y\right)\varphi(x) = 0\,, \quad y = y_j\,, \quad j = 1,2\,. \end{aligned} \tag{6}$$

Note that the Dirichlet and Neumann boundary conditions are obtained from (4) as special cases corresponding to $(a_j, b_j) = (1, 0)$ and $(a_j, b_j) = (0, 1)$ respectively. Our main interest in the present paper is to investigate the vacuum expectation values (VEV's) of the energy-momentum tensor for the field $\varphi(x)$ in the region $z_1 < z < z_2$. The presence of boundaries modifies the spectrum of the zero-point fluctuations compared to the case without boundaries. This results in the shift in the VEV's of the physical quantities, such as vacuum energy density and stresses. This is the well-known Casimir effect.

It can be shown that for a conformally coupled scalar by using the field equation (1) the expression for the energy-momentum tensor can be presented in the form [36]

$$T_{\mu\nu} = \nabla_\mu\varphi\nabla_\nu\varphi - \xi\left[\frac{g_{\mu\nu}}{D-1}\nabla_\rho\nabla^\rho + \nabla_\mu\nabla_\nu + R_{\mu\nu}\right]\varphi^2\,, \tag{7}$$

where $R_{\mu\nu}$ is the Ricci tensor. The quantization of a scalar field on a background of metric (2) is standard. Let $\{\varphi_\alpha(x), \varphi_\alpha^*(x)\}$ be a complete set of orthonormalized positive and negative frequency solutions to the field equation (1), obeying the boundary condition (4). By expanding the field operator over these eigenfunctions, using the standard commutation rules and the definition of the vacuum state for the vacuum expectation values of the energy-momentum tensor one obtains

$$\langle 0|T_{\mu\nu}(x)|0\rangle = \sum_\alpha T_{\mu\nu}\{\varphi_\alpha, \varphi_\alpha^*\}\,, \tag{8}$$

where $|0\rangle$ is the amplitude for the corresponding vacuum state, and the bilinear form $T_{\mu\nu}\{\varphi,\psi\}$ on the right is determined by the classical energy-momentum tensor (7). In the problem under consideration we have a conformally trivial situation: a conformally invariant field on a background of conformally flat spacetime. Instead of evaluating (8) directly on the background of the curved metric, the vacuum

[1] Further developments in the Casimir effect can be found in [35].

expectation values can be obtained from the corresponding flat spacetime results for a scalar field $\bar{\varphi}$ by using the conformal properties of the problem under consideration. Under the conformal transformation $g_{\mu\nu} = \Omega^2 \eta_{\mu\nu}$ the $\bar{\varphi}$ field will change by the rule

$$\varphi(x) = \Omega^{(1-D)/2} \bar{\varphi}(x) , \qquad (9)$$

where for the metric (2) the conformal factor is given by $\Omega = \mathrm{e}^{-\sigma(z)}$. The boundary conditions for the field $\bar{\varphi}(x)$ we will write in a form similar to (6):

$$\left(\bar{a}_j + (-1)^{j-1} \bar{b}_j \partial_z\right) \bar{\varphi} = 0 , \quad z = z_j , \quad j = 1, 2 , \qquad (10)$$

with constant Robin coefficients $\bar{a}_j$ and $\bar{b}_j$. Comparing to the boundary conditions (4) and taking into account the transformation rule (9) we obtain the following relations between the corresponding Robin coefficients:

$$\bar{a}_j = a_j + (-1)^{j-1} \frac{D-1}{2} \sigma'(z_j) \mathrm{e}^{\sigma(z_j)} b_j , \quad \bar{b}_j = b_j \mathrm{e}^{\sigma(z_j)} . \qquad (11)$$

Note that as the Dirichlet boundary conditions are conformally invariant the Dirichlet scalar in the curved bulk corresponds to the Dirichlet scalar in a flat spacetime. However, for the case of a Neumann scalar the flat spacetime counterpart is a Robin scalar with $\bar{a}_j = (-1)^{j-1}(D-1)\sigma'(z_j)/2$ and $\bar{b}_j = 1$. The Casimir effect with boundary conditions (10) on two parallel plates on a Minkowski spacetime background is investigated in [29] for a scalar field with a general conformal coupling parameter. In the case of a conformally coupled scalar the corresponding regularized VEV's for the energy-momentum tensor are uniform in the region between the plates and have the form

$$\left\langle \bar{T}^{\mu}_{\nu} [\eta_{\alpha\beta}] \right\rangle_{\mathrm{ren}} = -\frac{J_D(B_1, B_2)}{2^D \pi^{D/2} a^{D+1} \Gamma(D/2+1)} \mathrm{diag}(1, 1, \ldots, 1, -D) , \quad z_1 < z < z_2 , \qquad (12)$$

where

$$J_D(B_1, B_2) = \mathrm{p.v.} \int_0^{\infty} \frac{t^D \mathrm{d}t}{\frac{(B_1 t-1)(B_2 t-1)}{(B_1 t+1)(B_2 t+1)} \mathrm{e}^{2t} - 1} , \qquad (13)$$

and we use the notation

$$B_j = \frac{\bar{b}_j}{\bar{a}_j a} , \quad j = 1, 2 , \quad a = z_2 - z_1 . \qquad (14)$$

For the Dirichlet and Neumann scalars $B_1 = B_2 = 0$ and $B_1 = B_2 = \infty$ respectively, and one has

$$J_D(0, 0) = J_D(\infty, \infty) = \frac{\Gamma(D+1)}{2^{D+1}} \zeta_{\mathrm{R}}(D+1) , \qquad (15)$$

with the Riemann zeta function $\zeta_{\mathrm{R}}(s)$. Note that in the regions $z < z_1$ and $z > z_2$ the Casimir densities vanish [29]:

$$\left\langle \bar{T}^{\mu}_{\nu} [\eta_{\alpha\beta}] \right\rangle_{\mathrm{ren}} = 0 , \quad z < z_1 , \quad z > z_2 . \qquad (16)$$

This can also be obtained directly from (12) taking the limits $z_1 \to -\infty$ or $z_2 \to +\infty$.

The vacuum energy-momentum tensor on a curved background (2) is obtained by the standard transformation law between conformally related problems (see, for instance, [36]) and has the form

$$\langle T^{\mu}_{\nu} [g_{\alpha\beta}] \rangle_{\mathrm{ren}} = \langle T^{\mu}_{\nu} [g_{\alpha\beta}] \rangle^{(0)}_{\mathrm{ren}} + \langle T^{\mu}_{\nu} [g_{\alpha\beta}] \rangle^{(b)}_{\mathrm{ren}} . \qquad (17)$$

Here the first term on the right is the vacuum energy-momentum tensor for the situation without boundaries (gravitational part), and the second one is due to the presence of boundaries. As the quantum field is conformally coupled and the background spacetime is conformally flat the gravitational part of the energy-momentum tensor is completely determined by the trace anomaly and is related to the divergent part of the corresponding effective action by the relation [36]

$$\langle T^{\mu}_{\nu} [g_{\alpha\beta}] \rangle^{(0)}_{\mathrm{ren}} = 2 g^{\mu\sigma}(x) \frac{\delta}{\delta g^{\nu\sigma}(x)} W_{\mathrm{div}}[g_{\alpha\beta}] . \qquad (18)$$

Note that in an odd number of spacetime dimensions the conformal anomaly is absent and the corresponding gravitational part vanishes:

$$\langle T^{\mu}_{\nu} [g_{\alpha\beta}] \rangle^{(0)}_{\mathrm{ren}} = 0 , \quad \text{for even } D . \qquad (19)$$

The boundary part in (17) is related to the corresponding flat spacetime counterpart (12) and (16) by the relation [36]

$$\langle T^{\mu}_{\nu} [g_{\alpha\beta}] \rangle^{(b)}_{\mathrm{ren}} = \frac{1}{\sqrt{|g|}} \langle \bar{T}^{\mu}_{\nu} [\eta_{\alpha\beta}] \rangle_{\mathrm{ren}} . \qquad (20)$$

By taking into account (12) we therefore obtain

$$\langle T^{\mu}_{\nu} [g_{\alpha\beta}] \rangle^{(b)}_{\mathrm{ren}} = -\frac{\mathrm{e}^{(D+1)\sigma(z)} J_D(B_1, B_2)}{2^D \pi^{D/2} a^{D+1} \Gamma(D/2+1)} \mathrm{diag}(1, 1, \ldots, 1, -D) , \qquad (21)$$

for $z_1 < z < z_2$, and

$$\langle T^{\mu}_{\nu} [g_{\alpha\beta}] \rangle^{(b)}_{\mathrm{ren}} = 0, \text{ for } z < z_1, z > z_2 . \qquad (22)$$

In (21) the constants B_j are related to the Robin coefficients in the boundary condition (4) by the formulae (14) and (11) and are functions of z_j. In particular, for Neumann boundary conditions $B_j^{(N)} = 2(-1)^{j-1}/[a(D-1)\sigma'(z_j)]$.

3 Surface energy tensor and branes tension

The total bulk vacuum energy per unit physical hypersurface on the brane at $z = z_j$ is obtained by integrating over the region between the plates:

$$E_j^{(b)} = \mathrm{e}^{D\sigma(z_j)} \int_{z_1}^{z_2} \langle T^0_0 \rangle^{(b)}_{\mathrm{ren}} \mathrm{e}^{-(D+1)\sigma(z)} \mathrm{d}z = -\frac{J_D(B_1, B_2) \mathrm{e}^{D\sigma(z_j)}}{2^D \pi^{D/2} \Gamma(D/2+1) a^D} ; \qquad (23)$$

this result differs from the total Casimir energy per unit volume, and the reason for this difference should be the existence of an additional surface energy contribution to the volume energy. The corresponding energy density is defined by the relation [29]

$$T_{00}^{(\mathrm{surf})} = -\frac{4\xi - 1}{2}\delta(z;\partial M)\varphi\partial_z\varphi\,, \tag{24}$$

located on the boundaries $z = z_j$, $j = 1, 2$, where now

$$\delta(z;\partial M) = \delta(z - z_2 - 0) - \delta(z - z_1 + 0)\,, \tag{25}$$

where $\delta(z - z_j \pm 0)$ is a one sided δ-distribution. In the general case (general coupling), the stress-energy tensor diverges close to the branes. This would also be expected in the conformal case if the branes are curved [34]. But in our case from the above formula it follows that the surface term is zero for Dirichlet or Neumann boundary conditions (as the factors φ or $\partial_z\varphi$ would then vanish) but yields a non-vanishing contribution for Robin boundary conditions. The corresponding VEV can be evaluated by the standard method explained in [29]. This leads to the formula

$$\begin{aligned} &\langle 0 \left| T_{00}^{(\mathrm{surf})} \right| 0\rangle \\ &= \frac{4\xi - 1}{2}\delta(z;\partial M)\left(\partial_z\langle 0\left|\varphi(z)\varphi(z')\right|0\rangle\right)|_{z'=z}\,, \end{aligned} \tag{26}$$

which provides the energy density on the plates themselves. The integrated surface energy per unit area is given by

$$\varepsilon_c^{(\mathrm{surf})} = \frac{1}{a}\int_{z_1}^{z_2} \mathrm{d}z\,\langle 0\left|T_{00}^{(\mathrm{surf})}\right|0\rangle\,, \tag{27}$$

where $a = z_2 - z_1$. After regularization for the surface energy per unit area one obtains

$$\bar{\mathrm{E}}^{(\mathrm{surf})} = a\varepsilon_c^{(\mathrm{surf})} = \sum_{j=1}^{2} E^{(\mathrm{s})(\mathrm{surf})}(\beta_j) - aD(4\xi - 1)\varepsilon_c^{(2)}\,, \tag{28}$$

with $\varepsilon_c^{(2)}$ defined as in following notation:

$$\begin{aligned} \varepsilon_c^{(2)} = &\frac{B_1 + B_2}{2^D \pi^{D/2} a^{D+1}\Gamma\left(1 + \frac{D}{2}\right)} \\ &\times \mathrm{p.v.}\int_0^\infty \mathrm{d}t \\ &\times \frac{t^D(1 - B_1B_2t^2)}{(1 - B_1t)^2(1 - B_2t)^2\,\mathrm{e}^{2t} - (1 - B_1^2t^2)(1 - B_2^2t^2)}\,. \end{aligned} \tag{29}$$

For Dirichlet ($B_1 = B_2 = 0$) and Neumann ($B_1 = B_2 = \infty$) scalars this term vanishes. Note that, as it follows from (28), the quantity $\varepsilon_c^{(2)}$ is the additional (to a single plate) surface energy per unit volume in the case of the conformally coupled scalar field.

As follows from (27), in our conformally curved background the surface energy per unit area located on the branes is given by

$$E_j^{(\mathrm{surf})} = \mathrm{e}^{D\sigma(z_j)}\bar{E}^{(\mathrm{surf})}\,. \tag{30}$$

As one can see from (28) the vacuum energy per unit hypersurface on the brane $z = z_j$ can contain terms in the form $\sum_{j=1}^{2} E^{(\mathrm{s})(\mathrm{surf})}(\beta_j)$ with constants β_1 and β_2, corresponding to the single brane contributions when the second brane is absent. Adding these terms to the vacuum energy corresponds to finite renormalization of the tension on both branes.

4 Casimir surface energy on the branes in AdS_{4+1} bulk and the cosmological constant problem

As an application of the general formulae from the previous section here we consider the important special case of the AdS_{4+1} bulk for which

$$\sigma = \ln(k_4 z) = k_4 y\,, \tag{31}$$

with $1/k_4$ being the AdS curvature radius. AdS space is a spacetime that has a maximal symmetry and a negative constant curvature, supported by a negative cosmological constant. For a 4+1-dimensional AdS space, the curvature radius is related to the cosmological constant by

$$k_4 = \left(\frac{-\Lambda}{6}\right)^{1/2}\,. \tag{32}$$

Now the expressions for the coefficients B_j, $j = 1, 2$ take the form

$$B_j = \frac{b_j k_4 z_j}{(z_2 - z_1)\left[a_j + 3(-1)^{j-1}k_4 b_j/2\right]}\,. \tag{33}$$

Note that the ratio z_2/z_1 is related to the proper distance between the branes Δy by the formula

$$z_2/z_1 = \mathrm{e}^{k_4\Delta y}\,, \quad \Delta y = y_2 - y_1\,. \tag{34}$$

For the surface energy per unit area located on the branes one has

$$E_j^{(\mathrm{surf})} = (k_4 z_j)^4\bar{E}^{(\mathrm{surf})}\,. \tag{35}$$

Then using (28), (29) and (30) the surface energy per unit area of branes in the AdS_{4+1} bulk is given by

$$\begin{aligned} \mathrm{E}^{(\mathrm{surf})} = &\frac{\Lambda^2 z_j^4}{36}\left(\sum_{j=1}^{2} E^{(\mathrm{s})(\mathrm{surf})}(\beta_j)\right. \\ &+ \frac{B_1 + B_2}{16\pi^2 a^4\Gamma(3)} \\ &\times \mathrm{p.v.}\int_0^\infty \mathrm{d}t \\ &\left.\times \frac{t^4(1 - B_1B_2t^2)}{(1 - B_1t)^2(1 - B_2t)^2\mathrm{e}^{2t} - (1 - B_1^2t^2)(1 - B_2^2t^2)}\right)\,. \end{aligned} \tag{36}$$

For two 3-branes with brane tension σ_0, the effective 4-dimensional cosmological constant as seen by an observer on the brane is taken to be zero, in other words, for a certain choice of Robin coefficients, one could make this vanish,

$$\Lambda_{\text{eff}} = \sigma_0 + E^{(\text{surf})}_{(\text{our brane})}(\beta) - \sqrt{\frac{6\Lambda^2}{\kappa^2}} = 0\,, \qquad (37)$$

where κ^2 is the 5-dimensional gravitational coupling, and Λ is the bulk cosmological constant. However, requiring (37) to cancel is still a fine-tuning. Then in our model the boundary condition is another possibility to make the cosmological constant vanish. We could obtain this result only in our case of interest (massless conformally case with general Robin boundary condition in odd-dimensional spacetimes).

Now we turn to the brane-world model introduced by Randall and Sundrum [19] and based on the AdS geometry with one extra dimension. The fifth dimension y is compactified on an orbifold, S^1/Z_2 of length Δy, with $-\Delta y \leq y \leq \Delta y$. The orbifold fixed points at $y = 0$ and $y = \Delta y$ are the locations of two 3-branes. For the conformal factor in this model one has $\sigma = k_4|y|$. The boundary conditions for the corresponding conformally coupled bulk scalars have the form (6) with Robin coefficients $a_j/b_j = -c_j k_4$, where the constants c_j are the coefficients in the boundary mass term [37]:

$$m_{\varphi}^{(b)2} = 2k_4\left[c_1\delta(y) + c_2\delta(y-\Delta y)\right]\,. \qquad (38)$$

Note that here we consider the general case when the boundary masses are different for different branes. Supersymmetry requires $c_2 = -c_1$. The surface energy per unit area on the branes in the Randall–Sundrum brane-world background are obtained from (36) with additional factor 1/2. This factor is related to the fact that now in the normalization condition for the eigenfunctions the integration goes over the region $(-\Delta y, \Delta y)$, instead of $(0, \Delta y)$. The coefficients B_j in the expression for $J_4(B_1, B_2)$ are given by the formula

$$B_j = -\frac{\mathrm{e}^{(j-1)k_4\Delta y}}{\mathrm{e}^{k_4\Delta y}-1}\frac{1}{c_j + (-1)^j 3/2}\,. \qquad (39)$$

Recently the energy-momentum tensor in the Randall–Sundrum brane-world for a bulk scalar with zero mass terms c_1 and c_2 is considered in [38]; see also [39].

5 Conclusion

The Casimir effect on two parallel plates in conformally flat brane-world geometries background due to a conformally coupled massless scalar field satisfying Robin boundary conditions on the plates is investigated. In the general case of Robin boundary conditions formulae are derived for the vacuum expectation values of the energy-momentum tensor from the corresponding flat spacetime results by using the conformal properties of the problem. The purely gravitational part arises due to the trace anomaly and is zero for an odd number of spacetime dimensions. In the region between the branes the boundary induced part for the vacuum energy-momentum tensor is given by (21), and the corresponding total bulk vacuum energy per unit hypersurface on the brane has the form (23). Further the surface energy per unit area located on the branes is given by (30). As an application of the general formula we have considered the important special case of the AdS_{4+1} bulk. In this specific example we can write the effective cosmological constant as (37), and for a certain choice of Robin coefficients, one could make the effective cosmological constant vanish. However, requiring (37) to cancel is still a fine-tuning. The surface energy is zero for Dirichlet or Neuman boundary conditions but yields a non-vanishing contribution for Robin boundary conditions. Moreover, there is a region in the space of Robin parameters in which the interaction forces between two 3-branes are repulsive for small distances and are attractive for large distances [21, 39]. This provides a possibility to stabilize the interplate distance by using the vacuum forces. Then maybe one can say that this kind of boundary condition is more natural for cosmology. On the other hand, one can think of many quantum effects that contribute similarly to the brane tension, the Casimir energy from fields confined on the brane, or the Casimir effect from other types of bulk field, which might play a role in realistic models. An application to the Randall–Sundrum brane-world model is discussed. In this model the coefficients in the Robin boundary conditions on branes are related to the boundary mass terms for the scalar field under consideration.

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